

Synthesis of a hybrid moving horizon observer for switching dynamic systems

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Abstract— This document suggests a synthesis of a hybrid observer for switching systems. Such a hybrid observer is made up of a discrete observer identifying the switching signal and another one producing an estimation of the evolution of the continuous state. The developed estimation technique is the moving-horizon-state estimation (MHSE). In fact, this approach enables us to transpose the problem of observation into a problem of optimization. The used optimization algorithm is the Levenberg-Marquardt method. The state estimation for SDC is liable to simultaneously reconstruct the switching signal and the continuous state vector. The illustrated method is validated through estimation problematic of a two-mode CSTR chemical reactor.

Keywords— hybrid observer, moving horizon, nonlinear hybrid systems, optimization algorithm.

I. INTRODUCTION

Currently, the systems gradually grow in complexity when presenting the continuous-characteristic variables and the eventually discrete variables. These so-called hybrid dynamic systems (HDS) are characterized by the interaction between the continuous and the discrete parts [1]-[2].

The process-control, control and surveillance (detection and defect diagnosis, break-downs in the system) require reliable knowledge of the state variables [3]. In most dynamic systems, the only available variables are the inputs and outputs. Therefore, it is often necessary to reconstruct the system state in order to develop the control [4]-[5]. Accordingly, we seek to synthesize an auxiliary system which can provide an estimate of the state and determine the unknown sizes. The synthesis is carried out throughout the measured signals and the set of knowledge, which can be collected, of the process [6].

The switching dynamic system (SDC) is a class of SDH. It is occasionally defined by the pair $(p; x)$ called the hybrid state vector which is a combination of the situation p (switching signal) indicating the active sub-system and the state value x [7]-[8].

The proposed hybrid observer consists of two parts: a discrete observer, based on the theory of discrete event systems which identifies the switching signal, and a

continuous observer, based on the classical theory of observers which produces an estimate of the evolution of the dynamic system continuous state [9]-[10]. The state estimation for the SDC is likely to simultaneously reconstruct the switching signal and the continuous state vector so as to provide an estimate of the hybrid state vector.

In this paper, we intend to build a hybrid observer that provides an identification of a mode in evolution \hat{p} and an estimate of the state vector \hat{x} . The developed estimation technique is the Moving Horizon State Estimation (MHSE). The very technique reformulates the estimation problem as a minimization of a criterion. In other words, it is meant to minimize the difference between the system measurement and its prediction on a moving horizon. In the schema of observation that we propose, we will use the Levenberg-Marquardt algorithm as an optimization routine.

The paper is organized as follows. In Section II, the estimation problem is formulated. The MHSE estimation algorithms are proved in Section III regarding the requirements for observability. In Section IV, to show the executions of the algorithms, this approach and the tools for the states reconstruction fall over the estimation problematic of a two-mode CSTR chemical reactor. Finally, a conclusion is given in Section V.

II. PROBLEM FORMULATION

We consider the switching system made up of N nonlinear sub-system described by the following equation:

$$\begin{cases} \dot{x} = f_{p(t)}(x(t)) + g_{p(t)}(x(t))u \\ y_m = h_m(x) \end{cases} \quad (1)$$

$p(t) \in I = 1, 2, \dots, N$ the index of the active mode, it also represents the discrete state of the hybrid system at moment t .

$p(t)$ can be related to a temporal criterion with regions or surfaces in the space of the state or with an external parameter [11]. $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control input vector, $y \in \mathbb{R}^m$ is the system output. We may define $f(\cdot)$,

$g(\cdot)$ and $h(\cdot)$ as the fields of vector that are assumed to be differentiable. f_p is a smooth function, $f_p(0) = 0$ for all $p \in I$ [12]-[13]. In this work, we consider only the phenomenon of Non-Zenon (in an interval of finite time, the number of switches is finite) [13].

We may note that $t_k, k = 1, 2, \dots$ is the k th instant of switches, $t_0 = 0$. For all $t_k \leq t < t_{k+1}$, $p(t) = p(t_k) = i$ with $i \in I$; i.e. the mode i is active on the interval $[t_k, t_{k+1}]$. The problem of observation can then be formulated in the following way: the mixed structure of SDH is the structure of the proposed hybrid observer which is a combination of a discrete observer and a continuous observer. In the discrete case, the observer is a detector of mode; it is the observer with discrete moving horizon using the routine of optimization of Levenberg-Marquardt. By using the data (y, u) , the discrete observer provides $\hat{p} \in \{1, 2, \dots, N\}$. Thereafter, this information \hat{p} as well as the data (y, u) are used by the continuous observer to build the continuous state vector \hat{x} [14]-[15].

In this work, the method of observation sits on the technique of observation with a moving horizon together with the routine of optimization of Levenberg-Marquardt.

III. SYNTHESIS OF THE HYBRID OBSERVER

The suggested hybrid observer is composed of a discrete observer to identify the current mode and a continuous observer to estimate the continuous state.

A. Presentation of the considered observer ¶

In the structure of the considered observer, the discrete observer uses, on the one hand, the continuous input u and, on the other hand, the output of the SDH model (fig.1). Regarding the continuous observer, it estimates the continuous state by using the estimate of the discrete state as well as the continuous input u and the continuous output.

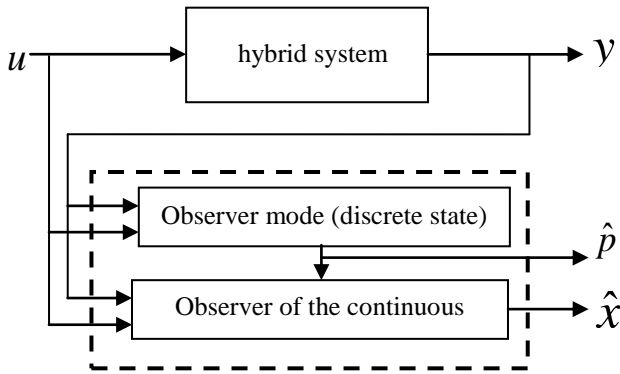


Fig. 1 hybrid observer

In each discrete mode, the continuous state of the considered SDC is written as:

$$\begin{cases} \dot{x} = f_p(x(t)) + g_p(x(t))u \\ y = h(x) \end{cases} \text{ For } p \in 1, 2, \dots, N \quad (2)$$

Concerning the mixed structure of SDH, the structure of the suggested observer is a combination of a discrete observer and a continuous observer. The considered hybrid observer is indicated on fig.1.

In this case, the discrete observer is a detector of mode. By using the data (y, u) , the discrete observer provides $\hat{p} \in \{1, 2, \dots, N\}$. Thereafter, this information and the data (y, u) are used by the continuous observer to build the continuous state vector \hat{x} [16].

The studied observation technique is the observer with a moving horizon using Levenberg-Marquardt optimization routine. Such an observer with a moving horizon MHSE will be studied thoroughly in the following section.

B. Moving Horizon State Estimator

The MHSE method solves the problem of the state estimation of a dynamic system by a nonlinear optimization problem [17].

Let us say that $x(t, 0, x_0)$ is the solution of the system at every moment t through the initial condition ' x_0 ' and $y(t, 0, x_0)$ is the corresponding output.

The principle of the estimator with a moving horizon consists in estimating on a given horizon $[t_k, t_k + lh_0]$ the single initial state \hat{x}_0 which minimizes a quadratic criterion happening on the error of the output. We use, then, the process dynamic model to estimate the current state $\hat{x}(t_k + lh_0)$ from the initial state \hat{x}_0 which is previously estimated (Fig.2). In the following sampling period, the horizon of estimate is shifted one period, then we resume the procedure of estimation on the new interval $[t_{k+1}, t_{k+1} + lh_0]$ and so on [18]-[19].

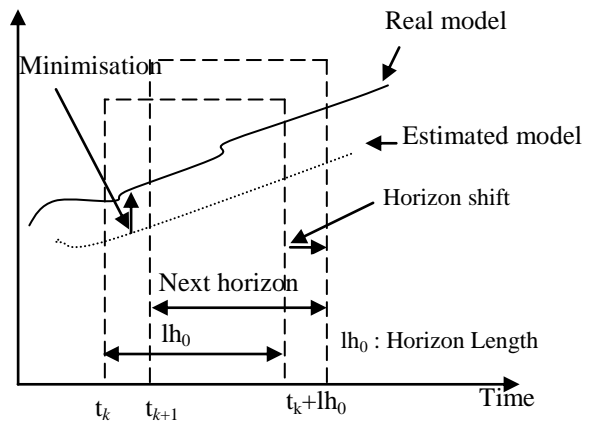


Fig. 2 MHSE Method

The base of the MHSE method is to transform the problem of estimate into a problem with quadratic criterion by using a window of a moving fixed size. Generally, the used criterion J is:

$$J(x) = \sum_{k=t_k}^{t_k+lh_0} (y_k - y_{m_k})^2 \quad (3)$$

Where y represents the measured outputs vector of the process, y_m is the output which is predicted from the observer and x is the vector of the states to be identified. t_k is the beginning of the horizon, lh_0 is the length of the horizon [20].

In this work, to solve this problem of optimization, we will use the algorithm of Levenberg-Marquardt for the continuous case and the discrete case.

C. Levenberg-Marquardt algorithm

The optimization algorithm of Levenberg-Marquardt (LM) is based on an update of Gauss-Newton type of the unknown factors. It is an iterative algorithm [21]-[22].

For the synthesis of the continuous observer, the equation (3)

admits an optimum if the condition of optimality ($\frac{\Delta J}{\Delta x} = 0$)

is verified.

By applying the limited development of this criterion, we have

$$\begin{aligned} J(x_{i+1}) &= J(x_i + \Delta x_i) \\ &= J(x_i) + \left[\frac{\partial J}{\partial x} \right]_{x=x_i}^T \Delta x_i \\ &\quad + \frac{1}{2} \Delta x_i^T \left[\frac{\partial^2 J}{\partial x \partial x_i} \right]_{x=x_i} \Delta x_i + 0 \|\Delta x\|^2 \end{aligned} \quad (4)$$

The variation ΔJ of the criterion is

$$\Delta J = Grad(x_i) \cdot \Delta x_i + \frac{1}{2} \Delta x_i^T Hess(i, j) \cdot \Delta x_i + 0 \|\Delta x\|^2 \quad (5)$$

With $Grad$ and $Hess$ are respectively the gradient and the hessian are given by

$$Grad(x_i) = \left[\frac{\partial J}{\partial x} \right]_{x=x_i}^T = -2 \sum_{k=1}^N (y_k - y_{m_k}(x)) \frac{\partial y_{m_k}(x_i)}{\partial x_i} \quad (6)$$

$$\begin{aligned} Hess(i, j) &= \left[\frac{\partial^2 J}{\partial x_j \partial x_i} \right] \\ &= 2 \sum_{k=1}^N \left(\frac{\partial y_{m_k}(x_i)}{\partial x_j} \right)^2 \\ &\quad - 2 \sum_{k=1}^N (y_k - y_{m_k}(x)) \frac{\partial y_{m_k}(x_i)}{\partial x_i \partial x_j} \end{aligned} \quad (7)$$

By applying the condition of optimality $\frac{\Delta J}{\Delta x} = 0$ we obtain

then:

$$Grad(x_i) + Hess(i, j) \cdot \Delta x_i = 0 \quad (8)$$

Thereafter, the variation of the state is given by

$$\Delta x_i = x_{i+1} - x_i = -Hess(i, j)^{-1} Grad(x_i) \quad (9)$$

The principle of the method of Levenberg-Marquardt consists in neglecting the term which can make the Hessian matrix negative and add a diagonal matrix to the Hessian in order to adjust the eigenvalues of the matrix of descent [23]-[24].

The new value of the state with the iteration $i+1$ is given by the equation of recurrence:

$$\hat{x}_{i+1} = \hat{x}_i - [Hess(i, j) + \lambda_i I]^{-1} \nabla J \quad (10)$$

$$\text{With } Hess(i, j) = 2 \sum_{k=1}^N \left(\frac{\partial y_{m_k}(x_i)}{\partial x_j} \right)^2 \quad (11)$$

λ_i is the relaxation coefficient. Adjust the eigenvalues of the hessian matrix by dividing or by multiplying once or many times until the convergence of the method [25].

For the design of the continuous observer, this technique of optimization is given by the algorithm 1.

Algorithm 1

Step 1 initialization of the parameters $k = 0$; λ ; ε_0

Step 2 calculation of $J(x_k), \nabla J(x_k), \nabla^2 J(x_k)$

Step 3 Tests on $\|\nabla J(x_k)\| < \varepsilon_0$

If yes the end and the convergence is obtained

If no we move to step 4

Step 4 Calculating $d_k = [\nabla^2 J(x_k) + \lambda I]^{-1} \cdot \nabla J(x_k)$

$$x_{0,k+1} = x_{0,k} + d_k \text{ and } J(x_{k+1})$$

Step 5 Tests on $J(x_{k+1}) < J(x_k)$

If yes calculate the new parameters $\lambda \leftarrow \lambda / 10$; $k \leftarrow k + 1$ and return to step 2.

If no calculate the new value of λ : $\lambda \leftarrow \lambda \times 10$ and return to step 4.

The state estimate x passes by the exploitation of the criterion to be minimized and by the algorithm of optimization.

Algorithm 2

For the design of the discrete observer, the same principle and the same technique of optimization are used for the estimate of the switching signal.

The quadratic criterion of the discrete state is given by:

$$\Phi = \sum_{k=t_k}^{sh_0} (S_k - S_{m_k})^2 \quad (12)$$

Where S represents the measured switching signal, this signal can be modeled by a temporal function describing the

trajectory taken by the modes i of the system. S_m is the switching signal predicted from the observer, t_k is the beginning of the horizon, lh_0 is the length of the horizon.

Algorithm 2 is similar to algorithm 1 by replacing the criterion J given by equation 3 by the criterion Φ given by equation 12.

D. General flow-chart of the hybrid observer MHSE

The general flowchart of the considered hybrid observer is a combination of two algorithms: the first gives an estimate of the continuous state x of the dynamic system (1) at the end of the chosen horizon, i.e. at time $l=t_k +lh_0$, but the second identifies the switching moment p . This algorithm uses the following terminology:

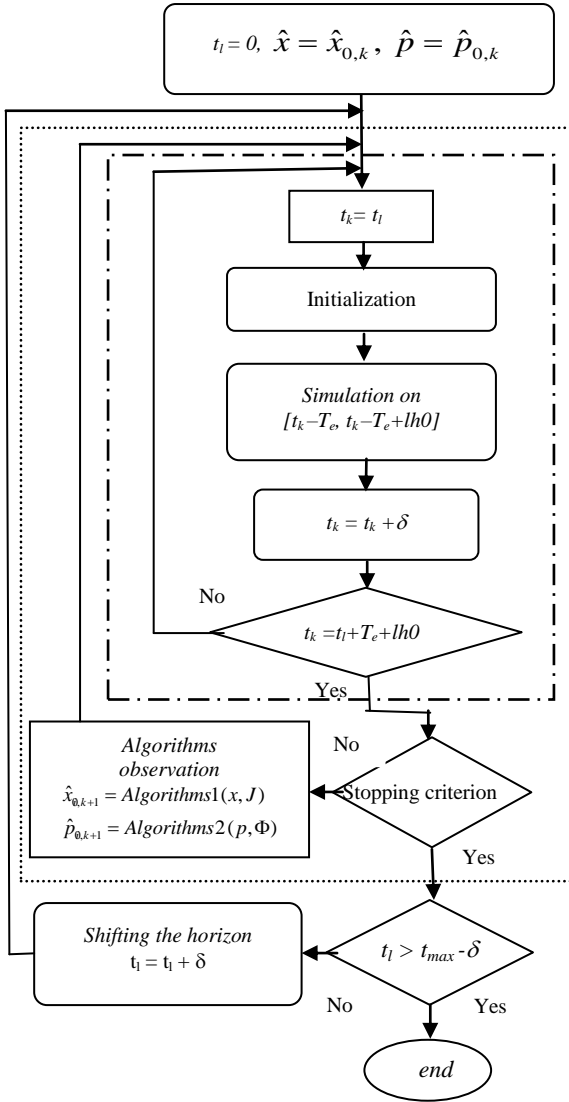


Fig. 3 Illustrative flow chart of the hybrid observer with a moving horizon

IV. APPLICATION TO A TWO-MODE CSTR CHEMICAL PROCESS

Continuous stirred tank reactors (CSTRs) are known to be one of the systems that exhibit complex behavior.

A. Description of the process

The considered process is a reactor with a continuously agitated tank (CSTR) where an irreversible exothermic reaction of the form $A \rightarrow B$ occurs. This system is supplied by an input current through a selected valve which is connected to two different sources: a source 1 supplies the component A with a flow q_1 , the concentration C_{Af1} and the temperature T_{f1} , and a source 2 supplies the component A with a flow q_2 , the concentration C_{Af2} and the temperature T_{f2} (fig. 4) [26]-[27]-[28].

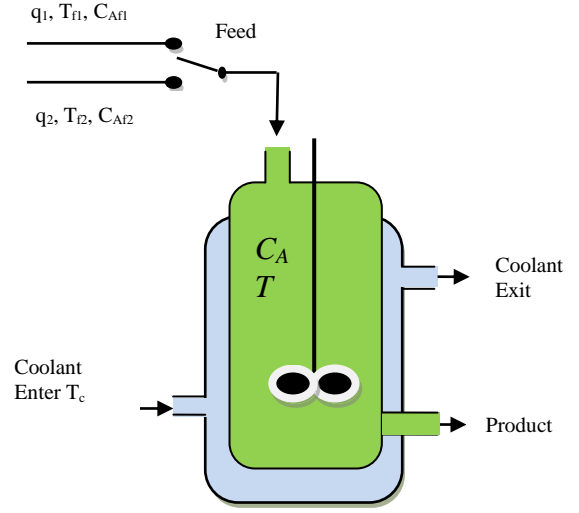


Fig.4 System of reactor with a continuously agitated tank (CSTR)

Every moment, the reactor is supplied by one of the sources. The reactor is, then, switched between two modes: mode 1 ($i = 1$) and mode 2 ($i = 2$) [29].

In each mode, the process is described by the following differential equations [29]:

$$\begin{cases} \frac{dC_A}{dt} = \frac{q_i}{V} (C_{Af_i} - C_A) - k_0 \exp\left(-\frac{E}{RT}\right) C_A \\ \frac{dT}{dt} = \frac{q_i}{V} (T_{f_i} - T) + \frac{(-Hr)}{\rho c_p} k_0 \exp\left(-\frac{E}{RT}\right) C_A \\ \quad + \frac{UA}{\rho \times c_p \times V} (T_c - T) \end{cases} \quad (13)$$

Where C_A is the concentration of the component A and T is the temperature of the reactor. The nominal values of the other parameters are given in table 1 [27]-[29]. The reactor is cooled by a flow of cooling liquid at a constant rate and a variable temperature T_c .

TABLE I
NUMERICAL PARAMETERS OF THE STUDIED REACTOR

Notation	Description	Value and unit
V	Reactor of the volume	100 L
K_0	Reaction rate constant	7.2 1010 L/min
E/R	Activation energy therm	8750 K
Hr	Enthalpy of reaction	-5.104 mol/m ³
ρ	Density of the fluid	1000 g/L
C_p	Heat capacity of the fluid	0.239 J/gK
U_A	Heat transfer constant	5.104 J/minK

The parameters describing the sources of supply of each mode are indicated in table 2 [29].

TABLE III
Parameters of the sources of supply

mode	$Q(l\ min^{-1})$	$C_{Af}(mol\ l^{-1})$	$T_c\ (K)$
1	50	1.5	350
2	200	0.75	350

B. Objective

During the reaction, we need to control the concentration C_A (mol/l) of component A in the reactional medium in the course of time. However, it is difficult to follow the evolution or the measurement of this state. It is then necessary to estimate the concentration C_A .

Similarly, we suppose that the position of the valve indicating the source of supply (active mode) is determined by an unknown arbitrary signal. Thus, a hybrid observer will be implemented for the reconstruction of the concentration C_A and for the determination of the active mode p (reconstructions of the switching signal). So, we re-enact the vector (\hat{x}, \hat{p}) .

C. Implementation of the estimator

In equation (13), we define the following vectors: $x = [x_1\ x_2]^T = [C_A\ T]^T$ is the model state vector; $y = T$ is the output; $u = T_c$ is the control.

We consider that the real switching signal $p(t)$ describing the active mode is approximately modeled by the following system of temporal equation:

$$p(t) = \begin{cases} 1 - e^{-10t} & \text{if } t \in [0, 0.5[\cup [1, 2[\cup [2.75, 4] & \text{mode 1} \\ 2 - e^{-20t} & \text{if } t \in [0.5, 1[\cup [2, 2.75[& \text{mode 2} \end{cases} \quad (14)$$

The evolution of the signal $p(t)$ is given by fig. 5

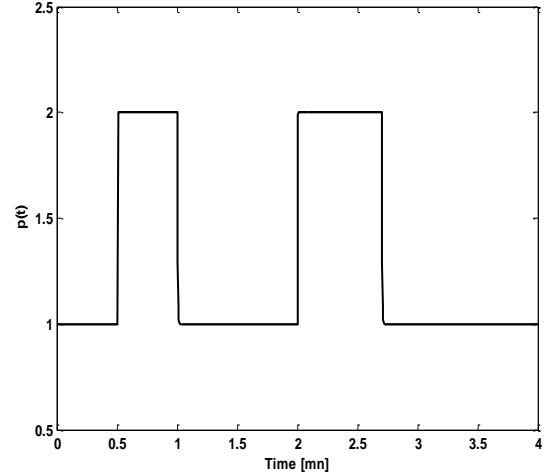


Fig. 5 The switching signal $p(t)$

The model was simulated according to the following initial conditions:

The state $x_0 = [3\text{mol}/l, 308\text{K}]$, the control $T_c = 350\text{K}$ and $p = p_0 = 1$ for the mode.

The initial state of the estimator is: $x_{m0} = [2\text{mol}/l, 308\text{K}]$ for the continuous state and $p_{m0} = 2$ for the mode.

In this work, for the continuous estimate, we are basically interested in the concentration C_A . Since the temperature T is the system output, so it is a measurement.

1) Simulation result:

The estimation results of the switching signal $p(k)$, the concentration C_A and the temperature T are respectively represented by fig. 6, fig.7 and fig. 8.

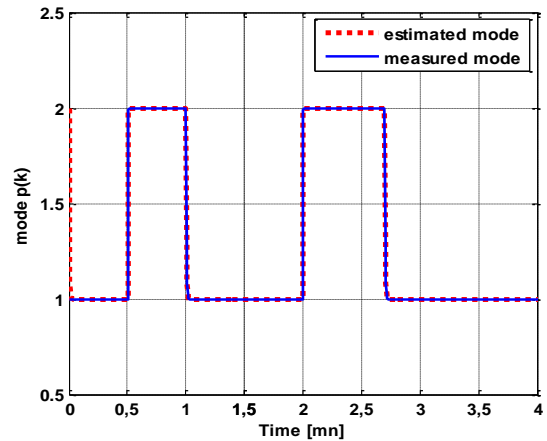


Fig. 6 Estimation of the mode p

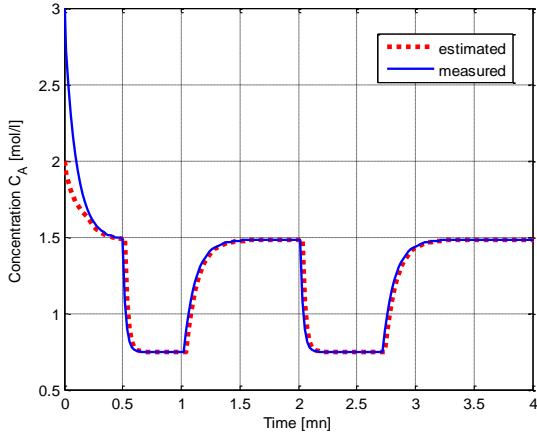


Fig.7 Estimation of the concentration C_A

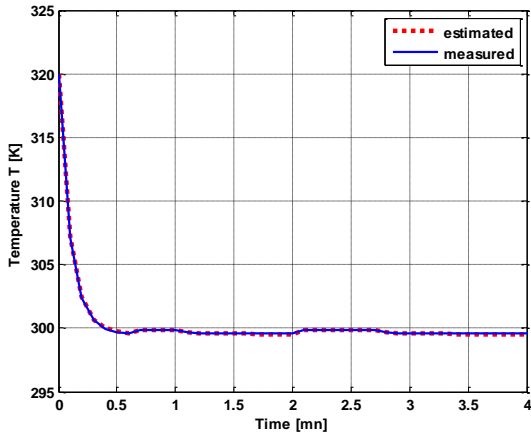


Fig.8 Estimation of the temperature T

Fig. 9 presents the estimation results of the temperature T in the presence of a Gaussian noise.

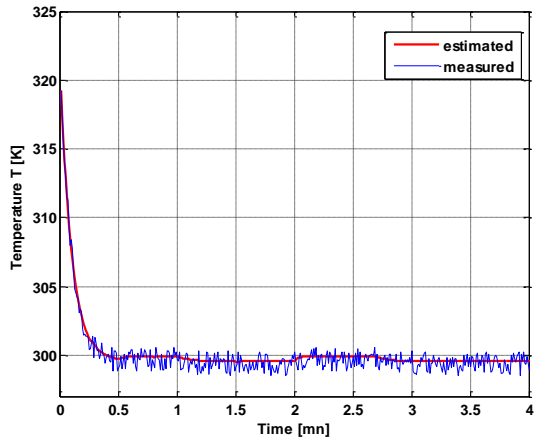


Fig. 9 Estimation of the temperature T in the presence of a Gaussian noise

2) Interpretation

The discrete observer re-enacts the switching signal \hat{p} during all the time of evolution with a good quality of estimation. The MHSE algorithm is likely to minimize the

error between the real value of the signal $p(k)$ and the estimated one (initially $\hat{p} = \hat{p}_0 = 2$) (fig.6). The continuous observer uses the switching signal which is identified as \hat{p} and the data u and y to determine the concentration \hat{C}_A . At every moment of switching i , the algorithm MHSE is liable to minimize the error between the simulated value C_A (initially 2 mol/l) and the estimated one (initially 3 mol/l) which will be null as from the minimal moments t_m (fig. 7). They are the necessary time for the estimator to eliminate all the errors between these two values in each switching of mode 1 towards mode 2. For the temperature, the estimate is ideal. Since the temperature is an output, so it is a measurement (fig. 8).

In each period, the estimator tends to reduce the difference between the real state and the estimated state until its disappearance. It is practical since this method is based on the approaches of optimization. Thus, the method finds the minimum and the states are well distinguished.

To test the robustness of the estimator with a moving horizon, a Gaussian noise was added to the measurement (fig.9). In the presence of the noise in the output T and for some amplitude, fig. 9 shows that the disturbance is quickly compensated and filtered by the MHSE method, the concentration is reconstructed and the observer can characterize its trajectory. Thus, the MHSE method keeps its promises and proves its power.

The perfect estimation is given by a hybrid observer which gives, via its discrete part, an identification \hat{p} of the mode which will be used by the continuous observer for the determination of the continuous state $\hat{x} = \begin{bmatrix} \hat{C}_A & \hat{T} \end{bmatrix}^T$. The approach of the recognized observation corrects the errors and updates the state of the process.

3) Observer performances

Our main contribution was to formalize an algorithm of nonlinear hybrid estimation that can be applied to a wide variety of systems. Our observer benefits from the advantages of the method with a moving horizon which is independent of the structures of the observed model. Our principal expectations were:

- The development of the combination of a continuous observer and a discrete observer to provide the hybrid vector (\hat{p}, \hat{x}) .
- The development of the coupling of a robust observer of the MHSE type with a technique of non-linear optimization.

The MHSE method is able to work with the models resulting from the various representations.

V. CONCLUSION

This document comes up with a new hybrid system observation technique: the hybrid observer with a moving horizon "MHSE". Such an observer is composed of a discrete

observer for the identification of the current mode and a continuous observer for the estimation of the continuous state. In fact, the MHSE method reformulates the problem of the state estimation of a dynamic system to a problem of nonlinear optimization. At the level of optimization, both observers, the continuous and the discrete, use the algorithm of Levenberg-Marquardt. The nonlinear MHSE hybrid observation was implemented and applied to the determination of the concentration of a species A in a continuously running reactor and to the identification of the switching signal. The obtained results show an excellent convergence between the simulated values and the results of the estimates. Equally important, the study of the quality of the estimate proves the strength of the MHSE estimator; in other words, its robustness with regard to the presence of the noise.

Unlike most of the estimation approaches existing in the literature, the technique of hybrid MHSE deals with the model independently of its structure since the estimator needs only one model of the system rather than to transform or linearize it.

The forthcoming researches will be purposed to direct the hybrid observation method towards the synthesis of an effective technique of diagnosis and the integration of the hybrid MHSE in a control law for the non linear industrial systems.

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